

Uniform Strategies in the Dynamic Epistemic Logic of Propositional Control

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joint work with

Tiago de Lima (Lens), Emiliano Lorini (Toulouse), Nicolas Troquard (Trento)

Workshop “Dynamics in Logic II”

Lille, March 1, 2012

An invitation to join www.sintelnet.eu

- “European Network for Social Intelligence” (SINTELNET)
 - FET open coordination action
 - coordination: David Pearce, Dirk Walther
- idea: revisit basic concepts of philosophy, humanities and social sciences in the light of new forms of information technology-enabled social environments
- actions:
 - working groups:
 - 1 action and agency (chair: Marek Sergot)
 - 2 interactive communication (chair: Andrew Jones)
 - 3 group attitudes (chairs: Andreas Herzig, Emiliano Lorini)
 - 4 socio-technical epistemology (chairs: Cristiano Castelfranchi, Luca Tummolini)
 - 5 social coordination (chair: Pablo Noriega)
 - interdisciplinary workshops
 - short term academic visits
 - production of guidelines and policy documents
- just started ⇒ join!

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Motivation

2 very different families of logics of action:

- ① $\langle \pi \rangle \varphi$ = “there is a possible execution of π after which φ ”
 - aim: prove correctness of programs
 - algorithmic logic [Salwicki 1970]
 - typically: dynamic logics [Pratt 1976; Parikh; Segerberg; . . .]
 - focus: both means (program π) and result (proposition φ)
- ② $\text{Stit}_i \varphi$ = “agent i sees to it that φ (whatever $-i$ does)”
 - focus: result of action
 - aim: clarify “being agentive for a proposition”
 - typically: stit logics
 - [von Kutschera, Belnap, Perloff, Horty, Wölfl, . . .]
 - embed Alternating-time Temporal Logic ATL
 - [Broersen, Herzig&Troquard 2007]
 - reasoning about uniform strategies: better than ATEL
 - [Herzig&Troquard 2006; Broersen et al. 2009; Herzig&Lorini2011]

⇒ relation? blend?

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Dynamic logic: advantages and shortcomings

advantages:

- means-end reasoning
- program operators
- standard possible worlds semantics

shortcomings:

- 1 no agents
 - action = event brought about by an agent
- 2 about opportunity rather than about action itself
 - $\langle \pi \rangle \varphi$ = “there is a **possible** execution of π such that. . .”
⇒ no reasoning about what I am actually doing
- 3 not suited for reasoning about actions in AI [McCarthy, Reiter]
 - no solution to the **frame problem**

1st idea: add agents to dynamic logic programs

- language:

$$\alpha ::= i:\pi_0 \mid \alpha; \alpha \mid \alpha \cup \alpha \mid \alpha^* \mid \varphi?$$

where π_0 is an atomic program and i is an agent

- semantics: an agent's action repertoire
[van der Hoek et al., AIJ 2005; Herzig et al., IJCAI 2011]

2nd idea: add a 'do' modality to dynamic logic

- language: another dynamic operator

[Cohen&Levesque 1990; Herzig&Lorini, JoLLI 2010; ...]

$\langle \alpha \rangle \varphi$ = “ \exists possible execution of α s.th. φ true afterwards”

$\langle\langle \alpha \rangle\rangle \varphi$ = “ α is going to be executed and φ is true afterwards”

- semantics: integrate linear time (histories)

3rd idea: solve the frame problem in dynamic logic

- language: atomic programs = propositional assignments
[van Ditmarsch, Herzig&de Lima, JLC 2011]

$$p := \varphi$$

⇒ frame axioms ‘built in’:

$$\models q \rightarrow \langle p := \varphi \rangle q \quad \text{for } p \neq q$$

- semantics: assignments **update models** (cf. DEL)

Outline

- 1 **DL-PC: language**
- 2 DL-PC: semantics
- 3 The Chellas stit
- 4 Relating DL-PC with the Chellas stit
- 5 Mathematical properties
- 6 Adding knowledge
- 7 Uniform strategies

Propositional Assignments

- propositional DL: “abstracts away from the nature of the domain of computation and studies the pure interaction between programs and propositions” [Harel et al. 2000]
 - abstract atomic programs
 - interpreted by accessibility relations
- first-order DL: assignments $x:=t$ of object variables to terms
 - example: $x:=x+1$
- here: assignments of propositional variables to truth values (“commands” [v. Eijck 2000])

$+p$ = “make p true”

$-p$ = “make p false”

Adding assignments to dynamic logic

[Tiomkin&Makowsky 1985; Wilm 1991; v. Eijck 2000]

- abstract programs plus assignments: two options
 - $\pm p$ modifies valuations of possible worlds globally
⇒ meaning?
 - $\pm p$ modifies valuations of possible worlds locally
⇒ undecidable [Tiomkin&Makowsky 1985]
- here: atomic programs = assignments [v.Eijck 2000]
 - no abstract programs
 - a single possible world is enough
 - model = valuation of classical propositional logic
 - small (interesting for model checking)

Language of DL-PC: assignments

- $\mathbb{P} = \{p, q, \dots\}$ = set of propositional variables
- assignments:
 - $+p$ = “ p becomes true”
 - $-p$ = “ p becomes false”
- $+P = \{+p : p \in P\}$
 - set of positive assignments of the variables in $P \subseteq \mathbb{P}$
- $-P = \dots$
 - \dots
- $\pm P = +P \cup -P$
 - $\pm p$ = arbitrary assignment from $+P \cup -P$

Language of DL-PC: actions and joint actions

- $\mathbb{A} = \{i, j, \dots\}$ = set of agents ('individuals')
- $\mathcal{JA} = \mathbb{A} \times \pm\mathbb{P}$ = set of all joint actions
 - $i:+p$ = "i makes p true"
 - $i:-p$ = ...
- group J 's part in joint action α :

$$\begin{aligned}\alpha_J &= \alpha \cap (J \times \pm\mathbb{P}) \\ &= \{i:\pm p \in \alpha : i \in J\}\end{aligned}$$

Language of DL-PC: action operators, agency operators

- executability of an action (opportunity):

$\langle \alpha \rangle \varphi$ = “each action in α may happen
and φ is true after the joint performance of α ”

- execution of an action:

$\langle\langle \alpha \rangle\rangle \varphi$ = “each action in α is going to occur
and φ is true after the joint performance of α ”

- being agentive for a proposition:

$\text{Stit}_J \varphi$ = “group J sees to it that φ ”

Language of DL-PC: formulas

φ	::=	p		
		\top		
		$\neg\varphi$		
		$\varphi \wedge \varphi$		
		$\langle\alpha\rangle\varphi$		opportunity of action
		$\langle\langle\alpha\rangle\rangle\varphi$		action
		$\text{Stit}_J\varphi$		agency
		$X\varphi$		temporal 'next'

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Valuations and their updates

- $\text{Val} \subseteq \mathbb{P}$ (valuation)
- update of Val by a joint action α
 - what if update by $\{i:+p, j:-p\}$?
 \Rightarrow don't change p 's truth value

$$\text{Val}^\alpha = (\text{Val} \setminus \{p : \exists i:-p \text{ in } \alpha \text{ and } \nexists j:+p \text{ in } \alpha\}) \cup \{p : \exists i:+p \text{ in } \alpha \text{ and } \nexists j:-p \text{ in } \alpha\}$$

Action repertoires and their updates

- $\text{Rep} \subseteq \mathbb{A} \times \pm\mathbb{P}$ (action repertoire)
 - Rep_i = agent i 's repertoire of actions
- joint action $\alpha \in \mathcal{JA}$ respects Rep iff $\alpha \subseteq \text{Rep}$
- update of Rep by joint action α :

$$\text{Rep}^\alpha = \text{Rep}$$

(but one may think of actions modifying repertoires)

Successor functions and their updates

- $\text{Succ} : \mathcal{JA}^* \longrightarrow \mathcal{JA}$ (successor function)
 - \mathcal{JA}^* = the set of all finite sequences of joint actions
 - $\text{Succ}(\sigma)$ = joint action that will be performed after the sequence of joint actions σ has occurred
 - $\text{Succ}(\text{nil})$ = joint action that is going to be performed now (nil = empty sequence)
- **update** of Succ by joint action α :

$$\text{Succ}^\alpha(\sigma) = \text{Succ}(\alpha \cdot \sigma)$$

($\alpha \cdot \sigma$ = composition of joint action α with sequence σ)

Models and their updates

- $M = (\text{Val}, \text{Rep}, \text{Succ})$ where
 - $\text{Val} \subseteq \mathbb{P}$ ('valuation')
 - $\text{Rep} \subseteq \mathcal{JA}$ ('repertoire')
 - $\text{Succ} : \mathcal{JA}^* \rightarrow \mathcal{JA}$ such that $\text{Succ}(\sigma)$ respects Rep, for all σ ('successor function')

- update of M by joint action α :

$$M^\alpha = (\text{Val}^\alpha, \text{Rep}^\alpha, \text{Succ}^\alpha)$$

Varying the successor function

- interpretation of Stit_J : quantify over the actions of $-J$ (“whatever the agents outside J do”)
- $\text{Succ} \sim_J \text{Succ}'$ iff for all σ , $(\text{Succ}(\sigma))_J = (\text{Succ}'(\sigma))_J$
 - “Succ and Succ’ agree on J ’s strategy”
- $M \sim_J M'$ iff $\text{Val} = \text{Val}'$, $\text{Rep} = \text{Rep}'$, and $\text{Succ} \sim_J \text{Succ}'$

Truth conditions

for $M = (\text{Val}, \text{Rep}, \text{Succ})$ a DL-PC model:

$$M \models \langle \alpha \rangle \varphi \quad \text{iff} \quad \alpha \subseteq \text{Rep} \text{ and } M^\alpha \models \varphi$$

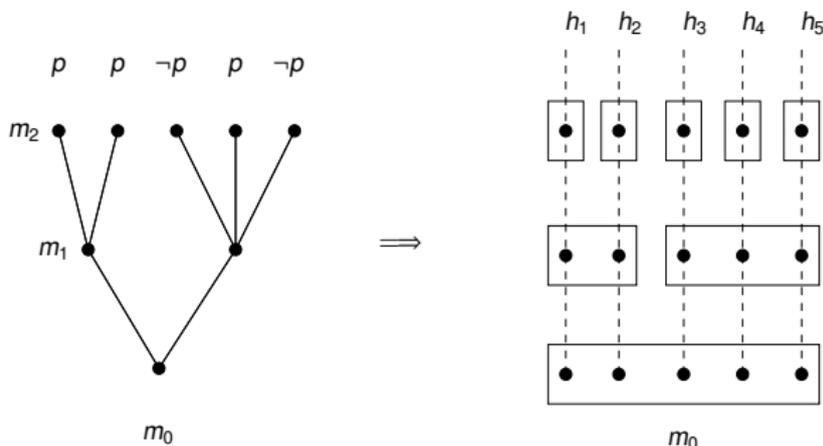
$$M \models \langle\langle \alpha \rangle\rangle \varphi \quad \text{iff} \quad \alpha \subseteq \text{Succ}(\text{nil}) \text{ and } M^\alpha \models \varphi$$

$$M \models \text{Stit}_J \varphi \quad \text{iff} \quad M' \models \varphi \text{ for every } M' \text{ such that } M \sim_J M' \\ \text{(keep } J\text{'s part in next joint action; vary } \neg J\text{'s part)}$$

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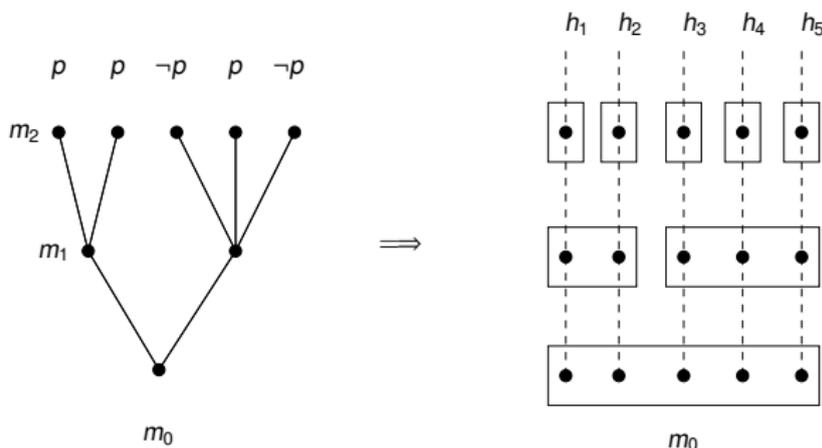
Models of the Chellas stit: branching time (BT)



discrete *BT* structure (*Mom*, $<$):

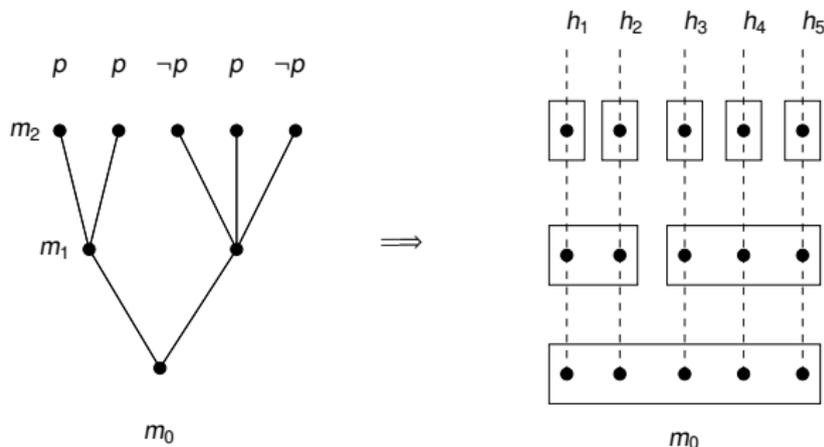
- set of moments *Mom*
- relation of temporal precedence $<$
 - **history** = maximally $<$ -ordered set of moments
 - *Hist* = set of all histories
 - *Hist*_{*m*} = set of histories passing through moment *m*
 - discrete: *succ*(*m*, *h*)

Models of the Chellas stit: agents' choices (AC)



- **Choice** : $\mathbb{A} \times Mom \longrightarrow Hist \times Hist$ such that each $Choice_i^m$ is an equivalence relation on $Hist_m$
 ($Choice_i^m$ = set of 'choice cells' for agent i at moment m)
- no choice between undivided histories: ...
- independence of agents: ...
- choice function can be extended to groups:
 $Choice_J^m = \bigcap_{i \in J} Choice_i^m$

BT+AC models



BT+AC model $\mathcal{M} = (Mom, <, Choice, v)$, where:

- $\langle Mom, < \rangle$ is a discrete branching time structure
- *Choice* is a choice function
- $v : (Mom \times Hist) \rightarrow 2^{\mathbb{P}}$ valuation function

The Chellas stit: truth conditions

formulas evaluated at a moment/history pairs m/h :

$$\mathcal{M}, m/h \models p \quad \text{iff} \quad p \in v(m/h)$$

$$\mathcal{M}, m/h \models \neg\varphi \quad \text{iff} \quad \dots$$

$$\mathcal{M}, m/h \models \varphi \wedge \psi \quad \text{iff} \quad \dots$$

$$\mathcal{M}, m/h \models \mathbf{X}\varphi \quad \text{iff} \quad \mathcal{M}, \text{succ}(m, h)/h \models \varphi$$

$$\mathcal{M}, m/h \models \text{Stit}_J\varphi \quad \text{iff} \quad \mathcal{M}, m/h' \models \varphi \text{ for all } h' \text{ s.th. } (h, h') \in \text{Choice}_J^m$$

- $\text{Stit}_J\varphi$ = “the alternative that is presently and actually chosen by J guarantees that φ is true”
 = “ J sees to it that φ ”

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Turning a DL-PC model into a $BT+AC$ model

for every DL-PC model $M = (\text{Val}, \text{Rep}, \text{Succ})$:

- $\text{Mom}_M = (2^{\text{Rep}})^*$ (finite sequences of joint actions)
- $\sigma <_M \sigma'$ iff $\sigma' = \sigma \cdot \sigma''$ for some $\sigma'' \neq \text{nil}$ (prefix relation)
 - history = infinite sequence of joint actions
 - Hist_σ = histories passing through moment σ
 $= \{h : \sigma \text{ is a prefix of } h\}$
- $\text{Choice}_i^\sigma = \{(h, h') : \text{there are } \alpha, \alpha' \text{ such that}$
 $\sigma \cdot \alpha \in h, \sigma \cdot \alpha' \in h', \text{ and } \alpha_i = \alpha'_i\}$
- recursive definition of valuation v_M :

$$v_M(\text{nil}, h) = \text{Val}$$

$$v_M(\sigma \cdot \alpha, h) = (v(\sigma, h))^\alpha$$

$(\text{Mom}_M, <_M, \text{Choice}_M, v_M)$ is a discrete $BT+AC$ model

The relation with the Chellas stit

- for DL-PC formulas φ without $\langle\langle\alpha\rangle\rangle$, $\langle\alpha\rangle$:
 - $M \models \varphi$ iff $(M, \text{Choice}, \nu), \text{nil}/h_M \models \varphi$
 where $h_M = (\text{nil}, \text{Succ}(\text{nil}), \text{Succ}(\text{Succ}(\text{nil})), \dots)$
 - if φ is valid in discrete $BT+AC$ models then φ is DL-PC valid

- converse does not hold:
 - $p \rightarrow \text{Stit}_i p$ valid in DL-PC, but not in $BT+AC$ models
 - $\text{Stit}_i(p \vee q) \rightarrow (\text{Stit}_i p \vee \text{Stit}_i q)$ valid in DL-PC, but not in $BT+AC$ models

- open question: are there schematic validities distinguishing DL-PC from $BT+AC$ models?

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Decision procedure (1)

simplify $\langle\langle.\rangle\rangle$:

$$\langle\langle\alpha\rangle\rangle\varphi \quad \leftrightarrow \quad \langle\alpha\rangle\varphi \wedge \langle\langle\alpha\rangle\rangle\top$$

$$\langle\langle\emptyset\rangle\rangle\top \quad \leftrightarrow \quad \top$$

$$\langle\langle\alpha \cup \beta\rangle\rangle\top \quad \leftrightarrow \quad \langle\langle\alpha\rangle\rangle\top \wedge \langle\langle\beta\rangle\rangle\top$$

Decision procedure (2)

simplify $\langle . \rangle$:

$$\langle \alpha \rangle p \quad \leftrightarrow \quad \begin{cases} \langle \alpha \rangle \top & \text{if } \exists i : i : +p \in \alpha \text{ and } \nexists j : j : -p \in \alpha \\ \perp & \text{if } \exists i : i : -p \in \alpha \text{ and } \nexists j : j : +p \in \alpha \\ \langle \alpha \rangle \top \wedge p & \text{if } \exists i, j : i : +p, j : -p \in \alpha \\ & \text{or } \forall i, j : i : +p, j : -p \notin \alpha \end{cases}$$

$$\langle \alpha \rangle \neg \varphi \quad \leftrightarrow \quad \langle \alpha \rangle \top \wedge \neg \langle \alpha \rangle \varphi$$

$$\langle \alpha \rangle (\varphi \wedge \psi) \quad \leftrightarrow \quad \langle \alpha \rangle \varphi \wedge \langle \alpha \rangle \psi$$

$$\langle \alpha \rangle \langle \beta \rangle \top \quad \leftrightarrow \quad \langle \alpha \rangle \top \wedge \langle \beta \rangle \top$$

\Rightarrow result: Boolean combination of **modal atoms**:

- propositional variables
- $\langle i : \pm p \rangle \top$
- $\overline{M} \langle \langle i : \pm p \rangle \rangle \top$, where \overline{M} is a sequence of $\langle \alpha \rangle$ and X

Decision procedure (3)

reduction axioms for Stit_J (cf. dynamic epistemic logics):

- $\text{Stit}_J(\varphi_1 \wedge \varphi_2) \leftrightarrow \text{Stit}_J\varphi_1 \wedge \text{Stit}_J\varphi_2$
- $\text{Stit}_J(p \vee \varphi) \leftrightarrow p \vee \text{Stit}_J\varphi$
 $\text{Stit}_J(\neg p \vee \varphi) \leftrightarrow \neg p \vee \text{Stit}_J\varphi$
- $\text{Stit}_J(\langle \alpha \rangle \top \vee \varphi) \leftrightarrow \langle \alpha \rangle \top \vee \text{Stit}_J\varphi$
 $\text{Stit}_J(\neg \langle \alpha \rangle \top \vee \varphi) \leftrightarrow \neg \langle \alpha \rangle \top \vee \text{Stit}_J\varphi$
- $\text{Stit}_J(\overline{M}\langle\langle i, \pm p \rangle\rangle \top \vee \varphi) \leftrightarrow \overline{M}\langle\langle i, \pm p \rangle\rangle \top \vee \text{Stit}_J\varphi$ if $i \in J$
 $\text{Stit}_J(\neg \overline{M}\langle\langle i, \pm p \rangle\rangle \top \vee \varphi) \leftrightarrow \neg \overline{M}\langle\langle i, \pm p \rangle\rangle \top \vee \text{Stit}_J\varphi$ if $i \in J$
- Let P and Q be two finite sets of modal atoms that are all of the form $\overline{M}\langle\langle i, \pm p \rangle\rangle \top$ with $i \notin J$. Then

$$\text{Stit}_J\left(\left(\bigvee P\right) \vee \neg\left(\bigwedge Q\right)\right) \leftrightarrow \begin{cases} \top & \text{if } P \cap Q \neq \emptyset \\ \neg \bigwedge_{\overline{M}\langle\langle i, \pm p \rangle\rangle \top \in Q} \langle i, \pm p \rangle \top & \text{if } P \cap Q = \emptyset \end{cases}$$

Decision procedure (4)

given a DL-PC formula φ :

- 1 take some innermost $\text{Stit}_J\psi$
- 2 transform ψ into a Boolean combination of modal atoms
- 3 eliminate Stit_J
- 4 iterate until no more agency operators Stit_J
 \Rightarrow result: $\varphi' =$ Boolean combination of modal atoms
- 5 call a SAT solver for $\varphi' \wedge (\bigwedge \Gamma_{\varphi'})$, where modal atoms are viewed as propositional variables and where

$$\Gamma_{\varphi'} = \{\overline{M}\langle\langle i:\pm p \rangle\rangle_{\top} \rightarrow \langle i:\pm p \rangle_{\top} : \\ \overline{M}\langle\langle i:\pm p \rangle\rangle_{\top} \text{ is a modal atom of } \varphi'\}$$

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Adding knowledge to DL-PC: models

- $M = (W, \{\approx_i\}_{i \in \mathcal{A}}, \{\text{Val}_w\}_{w \in W}, \{\text{Rep}_w\}_{w \in W}, \{\text{Succ}_{w \in W}\}_w)$ where
 - W set of possible worlds
 - $\approx_i \subseteq W \times W$, equivalence relation
 - $\text{Val}_w \subseteq \mathbb{P}$
 - $\text{Rep}_w \subseteq \mathcal{JA}$
 - $\text{Succ}_w : \mathcal{JA}^* \rightarrow \mathcal{JA}$ s.th. $\text{Succ}_w(\sigma) \subseteq \text{Rep}_w$ for all $\sigma \in \mathcal{JA}$

- constraints:
 - $\text{Succ}_w(\sigma) \subseteq \text{Rep}_w$, for all σ
 - if $w \approx_i w'$ then $(\text{Rep}_w)_i = (\text{Rep}_{w'})_i$
 - if $w \approx_i w'$ then $(\text{Succ}_w(\sigma))_i = (\text{Succ}_{w'}(\sigma))_i$, for all σ
 \Rightarrow will be valid:
 - $\langle \alpha \rangle \top \rightarrow \mathbb{K}_i \langle \alpha_i \rangle \top$
 - $\langle \langle \alpha \rangle \rangle \top \rightarrow \mathbb{K}_i \langle \langle \alpha_i \rangle \rangle \top$

Adding knowledge to DL-PC: truth conditions

$$\begin{array}{ll}
 M, w \models \langle\langle \alpha \rangle\rangle \varphi & \text{iff } \alpha \subseteq \text{Succ}_w(\text{nil}) \text{ and } M^{\langle\langle \alpha \rangle\rangle \top}, w \models \varphi \\
 M, w \models \langle \alpha \rangle \varphi & \text{iff } \alpha \subseteq \text{Rep}_w \text{ and } M^{\langle \alpha \rangle \top}, w \models \varphi \\
 M, w \models \text{Stit}_J \varphi & \text{iff } M', w \models \varphi \text{ for every } M' \text{ such that } M \sim_J M' \\
 M, w \models \mathbb{K}_i \varphi & \text{iff } M, w' \models \varphi \text{ for every } w' \text{ s.th. } w \approx_i w'
 \end{array}$$

update = announcement of executability/execution of α :

- $W^{\langle \alpha \rangle \top} = \{w \in W : \alpha \subseteq \text{Rep}_w\}$
- $W^{\langle\langle \alpha \rangle\rangle \top} = \{w \in W : \alpha \subseteq \text{Succ}_w(\text{nil})\}$

\Rightarrow Dynamic Epistemic Logic of Propositional Control (DEL-PC)

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Uniform strategies

- STIT plus knowledge better suited than ATEL
[Herzig&Troquard 2006, Broersen et al. 2009, Herzig&Lorini 2010]

- $\Box\varphi \stackrel{\text{def}}{=} \text{Stit}_\emptyset\varphi$ (historic necessity)
- $\Diamond\varphi \stackrel{\text{def}}{=} \neg\Box\neg\varphi$ (historic possibility)

- i knows that he can ensure φ :

$$K_i\Diamond\text{Stit}_i\varphi$$

- i knows how to ensure φ :

$$\Diamond K_i\text{Stit}_i\varphi$$

Uniform strategies: example

- hypotheses:
 - $\langle i: \neg p \rangle \top$ (*i* can make *p* false)
 - $K_i p$ (*i* knows that *p*)
 - $\neg K_i q \wedge \neg K_i \neg q$ (*i* uncertain about *q*)
- valid in DL-PC:
 - *i* knows that he can ensure that $p \leftrightarrow q$
 - $\models \text{Hypotheses} \rightarrow K_i \Diamond \text{Stit}_i X(p \leftrightarrow q)$
 - ... but *i* does not know how to ensure that $p \leftrightarrow q$
 - $\models \text{Hypotheses} \rightarrow \neg \Diamond K_i \text{Stit}_i X(p \leftrightarrow q)$

Conclusion

- DL-PC = PDL with assignments as the only atomic programs
 - complete axiomatisation
 - decidable (\neq group stit [Herzig&Schwarzenrüber])
 - with program operators:
 - Kleene star can be eliminated
 - SAT complexity: ExpTime complete
- DEL-PC = DL-PC plus epistemic operator
 - agents know what they are going to play
 - allows to reason about uniform strategies
 - t.b.d.: decidability & complexity of epistemic extension
- good basis for a logic of agent interaction
 - more elaborate account of constitutive rules: brute facts, institutional facts, roles [Herzig et al., CLIMA 2011]
 - social simulation [Gaudou et al. MABS 2011]